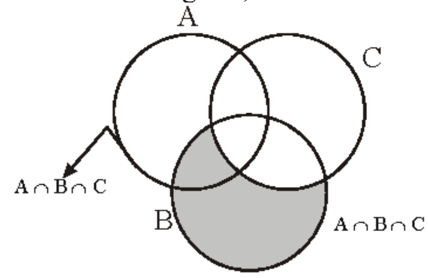
MATHEMATICS - H

1. Ans. B
2. Ans. A
3. Ans. C
   Required number of subsets $=8C_3 + 8C_4 + ...+8C_8 = 2^8 - 8C_0 - 8C_1 - 8C_2 = 256 - 1 - 8 - 28 = 219$
4. Ans. C
5. Ans. C
   $$x^3 - \frac{1}{x^3} + \left(\frac{1}{x}\right)^3 - \frac{1}{\left(\frac{1}{x}\right)^3}$$
   $$x^3 - \frac{1}{x^3} + \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}} \quad = x^3 \frac{1}{x^3} + \frac{1}{x^3} - x^{23} = 0$$
6. Ans. B
7. Ans. E
   (2 - k) (3 - k) - 2 = 0
   $6 - 5k + k^2 - 2 = 0$
   $k^2 - 5k + 4 = 0$
   (k - 4) (k - 1) = 0
   k = 4 or 1
   when k = 4
   $5k - k^2 = 20 - 16 = 4$
   when k = 1
   5 - 1 = 4
8. Ans. D
9. Ans. B
10. Ans. D
11. Ans. C
12. Ans. A
13. Ans. E
14. Ans D
    $$\lim_{x\to 5} \frac{x^k - 5^k}{x - 5} = k.5^{k-1}$$
    $$k.5^{k-1} = 4.5^3$$
15. Ans. C
16. Ans. B
    Applying L.H rule
17. Ans. C
18. Ans. D
19. Ans.A
    $y = e.e^{\log x}$
    $y = ex$
    $$\frac{dy}{dx} = e$$
20. Ans. E
    $$y = \tan^{-1}\left[\frac{2^x(2-1)}{1+2^x.2^{x+1}}\right] = \tan^{-1}\left[\frac{2^{x+1}-2^x}{1+2^x.2^{x+1}}\right]$$
    $$= \tan^{-1}\left(2^{x+1}\right) - \tan^{-1}\left(2^x\right)$$
    $$\frac{dy}{dx} = \frac{1}{1+2^{2(x+1)}}\,2^{x+1}\log 2 - \frac{2^x \log 2}{1+2^{2x}}$$
    $$\therefore \frac{dy}{dx} \text{ at } x = 0 = (\log 2)\left(\frac{2}{5} - \frac{1}{2}\right) = \log 2\left(-\frac{1}{10}\right)$$
21. Ans. A
22. Ans. A
23. Ans. B
    $$y = \sin^{-1} x$$
    $$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{2}\frac{(-2x)}{\left(1-x^2\right)^{3/2}}$$
    $$= \frac{1}{\left(1-x^2\right)}.\frac{x}{\sqrt{1-x^2}}$$
    $$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} = x - \frac{dy}{dx}$$
24. Ans. A
25. Ans. B
26. Ans. E
27. Ans. C
28. Ans. B
29. Ans. C
30. Ans. C
    $$2x\frac{dy}{dx} = y + 3 \Rightarrow 2\frac{dy}{y+3} = \frac{dx}{x}$$
    $\Rightarrow$ 2 log (y + 3) = log x + log c
    $\Rightarrow$ $(y + 3)^2 = cx$ which is a parabola
31. Ans. B
32. Ans. B
33. Ans. D
34. Ans. A
35. Ans. C
    Let $x_1, x_2, ...x_n$ be n observations.
    $\bar{x} = \frac{1}{n}\Sigma x_i$ and let $y_i = \frac{x_i}{\alpha} + 10$
    then $\frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{\alpha}\left(\frac{1}{n}\Sigma n_i\right) + \frac{1}{n}(10n)$
    $$\Rightarrow \bar{y} = \frac{1}{\alpha}\bar{x} + 10$$
36. Ans. B
37. Ans. A
    From Venn diagram, we can see that



$$P(B\cap C) = P(B) - P(A\cap B\cap \bar{C}) - P(\bar{A}\cap B\cap \bar{C})$$
$$= \frac{3}{4} - \frac{1}{3} - \frac{1}{3} = \frac{1}{12}$$

38. Ans. A
    Probability of getting head $= \frac{1}{2}$ and probability of throwing 5 or 6 with a dice $= \frac{2}{6} = \frac{1}{3}$. He starts with

a coin and alternately tosses the coin and throws the dice and he will win if he get a head before he get 5 or 6.

$\therefore \text{Probability} = \frac{1}{2} + \left(\frac{1}{2}.\frac{2}{3}\right).\frac{1}{2} + \left(\frac{1}{2}.\frac{2}{3}\right).\left(\frac{1}{2}.\frac{2}{3}\right) \times \frac{1}{2} + ..$

$$= \frac{1}{2}\left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + ...\right] = \frac{1}{2}.\frac{1}{1-(1/3)} = \frac{3}{4}$$

39. Ans : C
40. Ans : A
41. Ans. D

$$\frac{dy}{dx} = \frac{(\log y - x)}{y \log y}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{y \log y} = \frac{1}{y}$$

$$\therefore \text{I.F} = e^{\int \frac{1}{y \log y} dy} = e^{\log \log y} = \log y$$

42. Ans. D
43. Ans. C
44. Ans. C
45. Ans. B
46. Ans. A Using Baye's theorem

47. Ans.B
48. Ans. B
Put y = vx

$$v + \frac{xdv}{dx} = \frac{x.vx}{x^2 + v^2x^2} = \frac{v^2}{1+v^2}$$

$$\therefore \int \frac{1+v^2}{v^3} + \int \frac{dx}{x} = 0$$

$$\Rightarrow y = ce^{x^2/2y^2}$$

49. Ans : E
50. Ans : C, put $\theta = 0^\circ$ or 90°
51. Ans : A
52. Ans : C
53. Ans : B
54. Ans : C, $\frac{\tan 3\theta - 1}{\tan 3\theta + 1} = \sqrt{3}$

$$\Rightarrow \tan 3\theta = \frac{1-\sqrt{3}}{1+\sqrt{3}}, \Rightarrow \tan 3\theta = \tan 105^0$$

$$3\theta = n\pi + \frac{7\pi}{12}, \quad \theta = \frac{n\pi}{3} + \frac{7\pi}{36}$$

55. Ans : D
56. Ans : C, $\cos\left(\sin^{-1} 2x\sqrt{1-x^2}\right) = \frac{1}{9}$

$$\cos \cos^{-1}\sqrt{1-4x^2+4x^4} = \frac{1}{9}$$

$$1 - 2x^2 = \frac{1}{9}, \quad x^2 = \pm\frac{2}{3}$$

57. Ans : B, $x + y + z = xyz$, $\Rightarrow \frac{x+y+z}{xyz} = 1$
58. Ans : B
59. Ans : A, $\cos^{-1}\cos\left(2\pi - \frac{\pi}{3}\right) + \sin^{-1}\sin\left(2\pi - \frac{\pi}{3}\right)$

$$= \frac{\pi}{3} - \frac{\pi}{3} = 0$$

60. Ans. A
$x^2 + x - 2 = 0, x \neq 1$
$\Rightarrow$ x = 2 &1 but n = 1 does not satisfy the given equation. $\therefore$ x = -2 is the only solution.
61. Ans. A
62. Ans. D Let f(x) = $3x^2 + 7x + 10$ then minimum value of
f(x) = $ax^2 + bx + c$ is $f\left(\frac{-b}{2a}\right) = f\left(\frac{-7}{6}\right)$
63. Ans. D
Eight men can sit a round a table in 7! ways. Now there are 8 places for 4 women to sit such that no two women can sit together is $^8P_4$ ways
Total number of ways = 7! x $^8P_4$
64. Ans. A
Required number of ways are
$\frac{8!}{2!3!} \times \frac{5!}{4!} = 16,800$
65. Ans.C
The number of ways can be given as follows
2 bowlers and 9 other players = $^4C_2 \times {}^9C_9$
3 bowlers and 8 other players = $^4C_3 \times {}^9C_8$
4 bowlers and 7 other players = $^4C_4 \times {}^9C_7$
Hence required number of ways
= 6 x 1 + 4 x 9 + 1 x 36 = 78
66. Ans. C

$$\tan\theta = \frac{1 - \cos \pi/5}{\sin \pi/5} = \tan(\pi/10)$$

67. Ans. D
68. Ans. D
Cubing & equating real and imaginary parts
we get $\frac{x}{p} = p^2 - 3q^2$ ....(1)
& $\frac{y}{q} = q^2 - 3p^2$ ....(2)
(1) + (2)$\Rightarrow \frac{x}{p} + \frac{y}{q} = -2\left(p^2 + q^2\right)$
69. Ans. B taking mudule on both sides
$|z| = \left(\sqrt{8}\right)^{33} / 2^{49} = \sqrt{2}$

70. Ans. C
$T_{r+1} = {}^nC_r x^r$ for $(1 + x)^n$
Here the coeff. is ^nC_r.
Given $^nC_{r-1} = {}^nC_{r+3} \Rightarrow {}^{20}C_{r-1} = {}^{20}C_{r+3}$
$\Rightarrow$ r = 9
71. Ans. C
Coff. of T_5, T_6 , T_7 and in A.P for $(1 + x)^n$

ie., ${}^nC_4$, ${}^nC_5$ & ${}^nC_6$ an in A.P

$\Rightarrow 2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$\Rightarrow n = 7$ & 14

72. Ans. B

${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots + {}^nC_n = 2n - 1$

${}^{14}C_1 + {}^{14}C_2 + {}^{14}C_2 + {}^{14}C_3 + \ldots\ldots + {}^{14}C_{14} = 2^{14} - 1$

73. Ans. B

If the intercepted region is bisected at (α, β),

then equation of line is $\dfrac{x}{2\alpha} + \dfrac{y}{2\beta} = 1$

ie., $\dfrac{x}{6} + \dfrac{y}{8} = 1$

74. Ans. A

L_1 (8, - 9) = 2 × 8 + 3 × -9 - 4 = -15

L_2 (8, - 9) = 6 × 8 + 9 × -9 + 8 = -25

Same sign $\Rightarrow$ point lies on the same side

75. Ans. B

76. Ans. A

As the circle touching both the coordinate axis in 4th quadrant of radius 3 units, equation of circles $(x - 3)^2 + (y + 3)^2 = 3^2$

77. Ans. A

78. Ans. D

79. Ans. C Given ellipse is $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

$PF_1 + PF_2 = 2a = 10$

80.Ans. D Clearly, $e = \dfrac{2}{3}$ and $e' = \dfrac{3}{2}$ $\therefore ee' = 1$

81. Ans. B

given integral = $\displaystyle\int \frac{x\left(1 - \frac{1}{x^3}\right)^{1/4}}{x^5} dx$

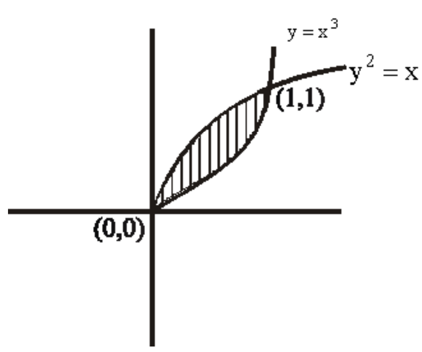
$= \displaystyle\int \left(1 - \frac{1}{x^3}\right) \cdot \frac{1}{x^4} dx$ put $1 - \dfrac{1}{x^3} = t$

82. Ans. B Put sinx = u

83. Ans. A

$$\int_0^a f(x)dx = \int_0^a f(a - x)dx$$

84. Ans. C



Area = $\displaystyle\int_0^1 \left(\sqrt{x} - x^2\right) dx = \frac{5}{12}$

85. Ans : B

For $0 \le x \le \dfrac{\pi}{4}, \sin x \le \cos x$ and hence

$\sqrt{1 - \sin\ 2x} = \sqrt{(\sin x - \cos x)^2}$

$= |\sin x - \cos x| = -(\sin x - \cos x)$

86. Ans : A, $\displaystyle\int 5^x 5^{5^x} 5^{5^{5^x}}\ dx$

$= \dfrac{1}{(\log 5)^3} \displaystyle\int (\log 5)^3\, 5^x 5^{5^x} 5^{5^{5^x}} dx$

$= \dfrac{5^{5^{5^x}}}{(\log 5)^3} + C$, $\because \dfrac{d}{dx}\left(5^{5^{5^x}}\right) = 5^{5^{5^x}} \log 5 \dfrac{d}{dx}\left(5^{5^x}\right)$

87. Ans : D, $\displaystyle\int_0^1 \sqrt{x(1 - x)}dx = \int_0^1 \sqrt{x - x^2} dx$

$= \displaystyle\int_0^1 \sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}\ dx$

$= \left[\dfrac{x - \frac{1}{2}}{2}\sqrt{x - x^2} + \dfrac{\frac{1}{4}}{2}\sin^{-1}\left(\dfrac{x - \frac{1}{2}}{\frac{1}{2}}\right)\right]_0^1$

$= \dfrac{1}{8}\sin^{-1}(1) - \dfrac{1}{8}\sin^{-1}(-1)$, $= \dfrac{1}{8}\left(\dfrac{\pi}{2} + \dfrac{\pi}{2}\right) = \dfrac{\pi}{8}$

88. Ans. A

$\text{amp.}\left(\dfrac{z - 1}{z + 1}\right) = \dfrac{\pi}{4} \Rightarrow \dfrac{\pi}{4}$ = amp (z-1) -amp(z +1)

$= \tan^{-1}\left(\dfrac{y}{x - 1}\right) - \tan^{-1}\left(\dfrac{y}{x + 1}\right)$

$\Rightarrow \dfrac{\pi}{4} = \tan^{-1}\left\{\left(\dfrac{y}{x - 1}\right) - \left(\dfrac{y}{x + 1}\right)\right\} \div \left\{1 + \left(\dfrac{y^2}{x^2 - 1}\right)\right\}$

$= \tan\dfrac{\pi}{4} = \dfrac{2y}{x^2 + y^2 - 1}$

$\Rightarrow x^2 + y^2 - 1 = 2y \Rightarrow x^2 + y^2 - 2y = 1$

89. Ans : B, First, we note that

$\dfrac{d}{dx}\left\{\displaystyle\int_0^x \phi(t)dt\right\} = \phi(x)$

$\therefore \displaystyle\lim_{x \to 0} \frac{\int_0^{x^2} \left(\sin\sqrt{t}\right)dt}{x^3}$

Let $\sqrt{t} = u$, then $t = u^2 \Rightarrow$ dt=2u du

90. Ans : B

91. Ans : D, $\displaystyle\int \tan x \sec^2 x\ dx = \frac{(\tan x)^2}{2}$

and also $\displaystyle\int \tan x \sec^2 x dx$

$= \displaystyle\int (\sec x)^1 (\sec x \tan x)dx = \frac{\sec^2 x}{2}$

So, correct alternative is (d) as $\dfrac{1}{2}(\cos x)^{-2}$ is also equal so $\frac{\sec^2 x}{2}$

92. Ans. C Let $\frac{x-1}{1}=\frac{y+3}{3}=\frac{z-2}{2}=\lambda(\text{say})$

$\therefore (x,y,z)=(\lambda+1.3\lambda-3,2\lambda+2)$

Using this in equation of plane

$\therefore 3(\lambda+1)-2(3\lambda-3)+(2\lambda+2)-7=0$

$\Rightarrow \lambda=4$

$\therefore$ (x,y,z) = the point of intersection of line and plane = (5,9,10)

93. Ans. B Angle between two planes will the angle between their normals.

If $\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$ then

$$\cos\theta=\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}=\frac{(i+2j+k)(-i+j+2k)}{|i+2j+k||-i+j+2k|}$$

$$\cos\theta=\frac{-1+2+2}{\sqrt{36}}=\frac{1}{2}\quad \therefore \theta=60^o$$

94. Ans. D

$$S.D=\frac{(\vec{b}_1\times\vec{b}_2)(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}$$

$$=\frac{(24\hat{i}+46\hat{j}+88\hat{k})(4\hat{i}+6\hat{j}+12\hat{k})}{\sqrt{(24)^2+(46)^2+(88)^2}}=\frac{714}{\sqrt{2609}}$$

95. Ans. C $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0, \vec{b}.\vec{c}=1/2$

$|\vec{a}\times\vec{b}|=|\vec{a}\times\vec{c}|=1$

$|\vec{a}\times\vec{b}-\vec{a}\times\vec{c}|^2=|\vec{a}\times\vec{b}|^2+|\vec{a}\times\vec{c}|^2 -2(\vec{a}\times\vec{b})(\vec{a}\times\vec{c})$

$$=1+1-2\begin{vmatrix}1 & 0\\ 0 & 1/2\end{vmatrix}=1$$

96. Ans. B

$|\vec{a}|=\sqrt{34}$

$|\vec{b}|=\sqrt{45}$

$|\vec{c}|=|\vec{a}\times\vec{b}|=39$

97. Ans. C

Let $\vec{u}=\vec{\alpha}+2\vec{\beta}$ and $\vec{v}=2\vec{\alpha}+\vec{\beta}$ then the diagonals lie along $\vec{u}+\vec{v}$ & $\vec{u}-\vec{v}$. Hence lengths of diagonals are given by $|\vec{u}+\vec{v}|^2=108$ and $|\vec{u}-\vec{v}|^2=4$

98. Ans. A

99. Ans : C

100. Ans : A

The plane passes through the points (4, 0, 0) and (0, 0, 3).

Any plane through (4, 0, 0) is

a(x-4) + b(y-0) + c(z-0) = 0 ..... (i)

$\because$ (0, 0, 3) lies in it

-4a + 3c = 0 ....(ii)

(i) is parallel to y axis

$\therefore$ a.0 + 1.b + 0.c = 0 $\Rightarrow$ b = 0

From (i) and (ii)

$a(x-4)+\frac{4}{3}az=0$ or 3x + 4z = 12

101. Ans : B

$$\begin{vmatrix}3-1 & k-(-1) & 0-1\\ 2 & 3 & 4\\ 1 & 2 & 1\end{vmatrix}=0$$

k = 9/2

102. Ans. D $\sin\theta=\frac{|5\times0-4\times1+7\times0|}{\sqrt{5^2+4^2+7^2}.\sqrt{1}}=\frac{4}{\sqrt{90}}$

103. Ans. A

$2\times\lambda-\lambda\times5+3\times-1=0$

$2\lambda-5\lambda-3=0$

$-3\lambda=3\quad \lambda=-1$

$\therefore \lambda^2+\lambda=(-1)^2+(-1)=0$

104. Ans.A

The required vector $\vec{r}=\lambda(\vec{a}+\vec{b}), \lambda$ is a scalar.

$\Rightarrow \vec{r}=\lambda\left(\frac{1}{9}(7\hat{i}-4\hat{j}-4\hat{k})+\frac{1}{3}(-2\hat{i}-\hat{j}+2\hat{k})\right)$

$=\frac{\lambda}{9}(\hat{i}-7\hat{i}+2\hat{k})$

Since, $|\vec{r}|^2=54$

$\Rightarrow \frac{\lambda^2}{81}(1+49+4)=54$

$\Rightarrow \lambda=\pm9$

Thus, the required vector is

$\vec{r}=\pm(\hat{i}-7\hat{j}+2\hat{k})$

105. Ans.A

The line is parallel to the plane since $(\hat{i}-\hat{j}+4\hat{k})(\hat{i}+5\hat{j}+\hat{k})=0$. Now, the distance (2,-2,3) from the plane x+5y+z-5= 0

ie, $\frac{|2-10+3-5|}{\sqrt{1+25+1}}=\frac{10}{3\sqrt{3}}$

106. Ans.B

Since, $(\vec{a}+\vec{b}+\vec{c})^2\geq0\Rightarrow 3+2\Sigma\vec{a}.\vec{b}\geq0$

or $-2\Sigma\vec{a}.\vec{b}\leq3$

$\therefore|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$

$=6-2\Sigma\vec{a}.\vec{b}\leq9$

107. Ans. C

108. Ans. C

109. Ans. B

110. Ans : C let x = $\sin\theta$

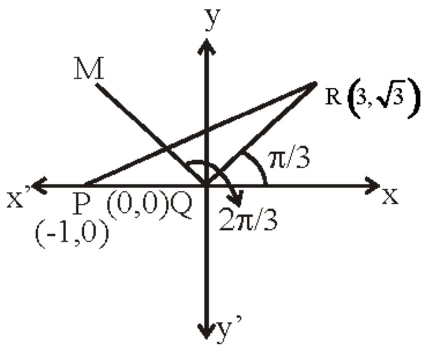
111. Ans. B Centre is the point of intersection of axes.

(ie., x + y - 2 = 0 and x - y = 0)

112. Ans. C

Now, slope of $QR=\frac{3\sqrt{3}-0}{3-0}=\sqrt{3}=\tan\theta$

$\Rightarrow \theta = \frac{\pi}{3}$



$\therefore$ The angle between $\angle PQR$ is $\frac{2\pi}{3}$, so that the line QM makes an angle $\frac{2\pi}{3}$ from positive direction of x-axis.

Slope of the line QM = $\tan\frac{2\pi}{3} = -\sqrt{3}$

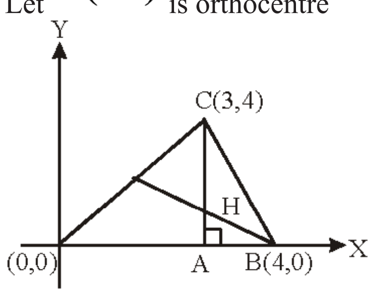
Hence, equation of the line QM is $y = -\sqrt{3}x$

or $\sqrt{3}x + y = 0$

113. Ans. D

114. Ans.D

Let $H(3,\alpha)$ is orthocentre



$\therefore$ Slope of BH $\times$ Slope of AC = -1

$= -\alpha.\frac{4}{3} = -1$

$\Rightarrow \alpha = \frac{3}{4}$

115. Ans.C

If the centroid is joined to the vertices, we get there triangle of equal area.

$\therefore R = G = \left(3, \frac{4}{3}\right)$

116. Ans. B

$2x^2 - 3x - 5 = 0$ $\therefore \alpha + \beta = 3/2$ $\alpha\beta = -5/2$

$\sigma_1 = 5/\alpha + 5/\beta = 5(\alpha+\beta)/\alpha\beta = -3$

and $\sigma_2 = 25/\alpha\beta = -10$

$\therefore$ Required equation is $x^2 + 3x - 10 = 0$

117. Ans. C

$Sn = \frac{n}{2}[2a + (n-1)d] = n^2p$

$\Rightarrow$ 2a + (n - 1)d = 2np ....(1)

$Sm = \frac{m}{2}[2a + (m-1)d] = m^2p$

$2a + (m - 1)d = 2m^2p$ ....(2)

(1) ...(2) $\Rightarrow$ (n - m)d = 2(n - m) p

d = 2p

(1) $\Rightarrow$ 2a + (n - 1)2p = 2np

$\Rightarrow$ 2a = 2p $\Rightarrow$ a = p

$Sp = \frac{p}{2}[2a + (p-1)d] = \frac{p}{2}\left[2p + (p-1)^2 p\right]$

$= \frac{p}{2}.2p \times p = p^3$

118. Ans. C

$T_o = 499$

$T_{499} = 9\left[T_p = q, T_q = p \Rightarrow T_{p+q} = 0\right]$

$\therefore T_{9+499} = 0$ ie., $T_{508} = 0$

119. Ans. C

120. Ans. D